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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

MATHEMATICS (SYLLABUS D) **4024/23**

Paper 2 **May/June 2010**

2 hours 30 minutes

Additional Materials: Answer Booklet/Paper, Electronic calculator, Geometrical instruments, Graph paper (2 sheets), Mathematical tables (optional)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Section A
Answer **all** questions.

Section B
Answer any **four** questions.

Show all your working on the same page as the rest of the answer.
Omission of essential working will result in loss of marks.
You are expected to use an electronic calculator to evaluate explicit numerical expressions. You may use mathematical tables as well if necessary.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

This document consists of **12** printed pages.

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[Turn over

Section A [52 marks]

Answer **all** questions in this section.

1 **(a)** Sarah bought some soup, apples and mushrooms from her local shop.
The table shows some of the amounts and prices.

Items	Price ($)
p cans of soup at 90 cents per can	6.30
1.5 kilograms of apples at $\$q$ per kilogram	4.35
r kilograms of mushrooms at $6.40 per kilogram	1.60

(i) Find the values of p, q and r. [2]

(ii) Sarah gives the shopkeeper $20.00 to pay for all these items.

How much change does she receive? [1]

(b)

Washing Machine

$980

<u>Finance offer</u>

Pay a 20% deposit
<u>and</u>
24 monthly payments of
$36 each

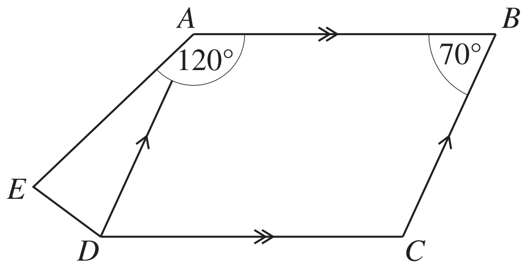
Lavin decides to buy this washing machine.

How much more would it cost Lavin if he paid for the washing machine using the finance offer instead of paying the $980 immediately? [2]

(c) Asif deposits $650 into a bank paying simple interest.
He leaves the money there for 5 years.
At the end of the 5 years, the amount in the bank is $763.75.

Calculate the percentage rate of interest the bank paid per year. [3]

2



The parallelogram *ABCD* forms part of the pentagon *ABCDE*.
$A\hat{B}C = 70°$ and $B\hat{A}E = 120°$.

(a) Find

(i) $B\hat{C}D$, [1]

(ii) $E\hat{A}D$. [1]

(b) $E\hat{D}C$ is twice $A\hat{E}D$.

Find

(i) $A\hat{E}D$, [3]

(ii) $E\hat{D}A$. [1]

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3 The mass and diameter of the planets in the inner solar system are shown in the table.

Planet	Mass (kg)	Diameter (km)
Mercury	3.30×10^{23}	4880
Venus	4.87×10^{24}	12 100
Earth	5.97×10^{24}	12 800
Mars	6.42×10^{23}	6790

(a) List the planets in order of mass, starting with the lowest. [1]

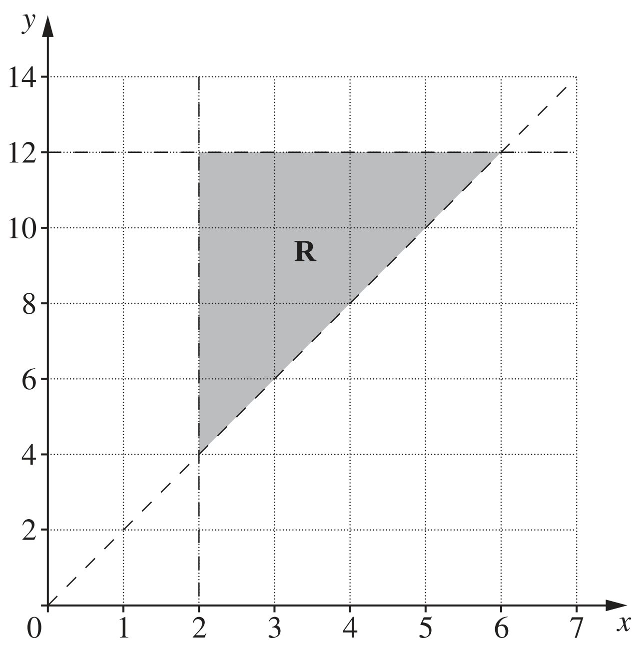
(b) Find the radius, in kilometres, of Mars, giving your answer correct to 1 significant figure. [1]

(c) Giving your answer in standard form, find the total mass, in kilograms, of Venus and Mars. [1]

(d) **[Volume of a sphere = $\frac{4}{3}\pi r^3$]**

Giving your answer in standard form, find the volume, in cubic kilometres, of the Earth. [2]

4



The shaded region, **R**, contained **inside** the dotted boundary lines, is defined by three inequalities.

(a) One of these inequalities is $x > 2$.

Write down the other two inequalities. [3]

(b) The points (c, d), where c and d are integers, lie in the shaded region **R**.

Find

(i) the maximum value of $c + d$, [1]

(ii) the value of d given that $d = 3c$. [1]

5 **(a)** Bertie goes shopping and buys three different types of fruit.
The first matrix below shows the number of kilograms of each fruit bought during two different weeks.
The second matrix shows the price per kilogram, in cents, of each fruit.

$$\begin{array}{l} \\ \text{Week 1} \\ \text{Week 2} \end{array} \begin{array}{c} \begin{array}{ccc} \text{bananas} & \text{apples} & \text{grapes} \end{array} \\ \begin{pmatrix} 1 & 2 & 0.5 \\ 1.5 & 1 & 1 \end{pmatrix} \end{array} \qquad \begin{array}{c} \text{price/kg} \\ \begin{pmatrix} 290 \\ 160 \\ 640 \end{pmatrix} \end{array} \begin{array}{l} \\ \text{bananas} \\ \text{apples} \\ \text{grapes} \end{array}$$

(i) $\mathbf{F} = \begin{pmatrix} 1 & 2 & 0.5 \\ 1.5 & 1 & 1 \end{pmatrix} \begin{pmatrix} 290 \\ 160 \\ 640 \end{pmatrix}$.

Find **F**. [2]

(ii) Explain the meaning of the information given by the matrix **F**. [1]

(iii) Find the total amount of money, in dollars, that Bertie spent on fruit during the two weeks. [1]

(b) The matrix **M** satisfies the equation

$$8\begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix} + 5\mathbf{M} = \mathbf{M}.$$

Find **M**. [2]

(c) $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$
$A = \{x : x \text{ is a multiple of } 3\}$
$B = \{x : x \text{ is a factor of } 24\}$
$C = \{x : x \text{ is an odd number}\}$

(i) Find

(a) $n(B)$, [1]

(b) $(A \cup B \cup C)'$. [1]

(ii) A number, k, is chosen at random from $\mathscr{E}$.

Find the probability that $k \in A \cap B$. [2]

6 **Answer the WHOLE of this question on a sheet of graph paper.**

The table below shows some values of x and the corresponding values of y for

$$y = \frac{2^x}{4}.$$

x	-1	0	1	2	3	4	5
y	m	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	n

(a) Calculate the values of m and n. [2]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-1 \leqslant x \leqslant 5$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $0 \leqslant y \leqslant 8$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to solve the equations

(i) $\frac{2^x}{4} = 3$, [1]

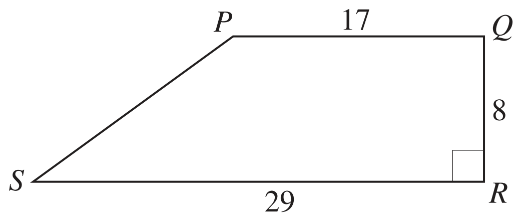
(ii) $2^x = 6$. [1]

(d) The equation $y = \frac{2^x}{4}$ can be written in the form $y = 2^t$.

(i) Find an expression for t in terms of x. [1]

(ii) Hence, find the equation of the line that can be drawn on your graph to evaluate y when $t = -\frac{3}{4}$. [1]

7 **(a)**



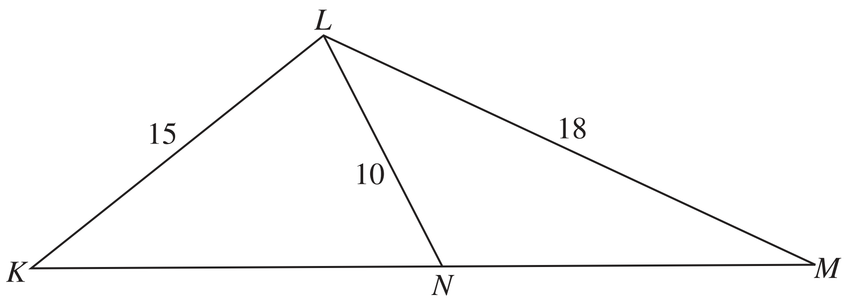
PQRS is a trapezium.
$PQ = 17$ cm, $QR = 8$ cm, $SR = 29$ cm and $S\hat{R}Q = 90°$.

Calculate

(i) the area of *PQRS*, [1]

(ii) $P\hat{S}R$. [2]

(b)



In the diagram, triangle *KLM* is similar to triangle *LNM*.
$KL = 15$ cm, $LM = 18$ cm and $LN = 10$ cm.

(i) Find *KM*. [2]

(ii) Find *KN*. [2]

(iii) *P* is the point on *LM* such that *PN* is parallel to *LK*.

Find $\dfrac{\text{the area of triangle } NPM}{\text{the area of trapezium } KLPN}$.

Give your answer as a fraction in its simplest form. [2]

Section B [48 marks]

Answer **four** questions in this section.

Each question in this section carries 12 marks.

8 Ahmed throws a ball to John.
The ball travels 10 metres at an average speed of x metres per second.

(a) Write an expression, in terms of x, for the time taken, in seconds, for the ball to travel from Ahmed to John. [1]

(b) John then throws the ball to Pierre.
The ball travels 15 metres.
The ball's average speed is 0.5 metres per second greater than the ball's average speed from Ahmed to John.

Write an expression, in terms of x, for the time taken, in seconds, for the ball to travel from John to Pierre. [1]

(c) The time taken between John catching the ball and then throwing it to Pierre is 2 seconds.
The total time taken for the ball to travel from Ahmed to Pierre is 7 seconds.

Write down an equation in x, and show that it simplifies to

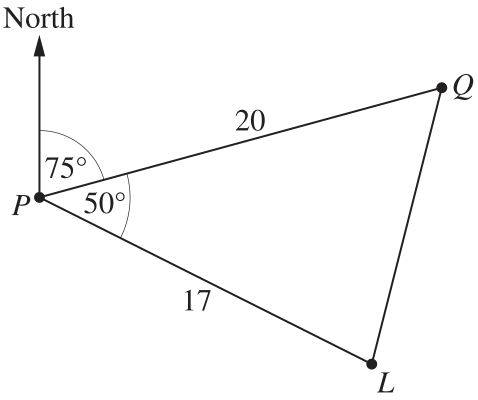
$$2x^2 - 9x - 2 = 0.$$ [3]

(d) Solve the equation $2x^2 - 9x - 2 = 0$, giving each answer correct to 2 decimal places. [4]

(e) **(i)** Find the average speed, in metres per second, of the ball as it travels from John to Pierre. [1]

(ii) How much longer does it take for the ball to travel from John to Pierre than from Ahmed to John?
Give your answer in seconds. [2]

9



The diagram shows two ports, P and Q, and a lighthouse L.
$PQ = 20$ km, $PL = 17$ km, $Q\hat{P}L = 50°$ and the bearing of Q from P is 075°.

(a) Find the bearing of P from L. [1]

(b) Calculate QL. [4]

(c) **(i)** Calculate $P\hat{L}Q$. [3]

(ii) Hence find the bearing of Q from L. [1]

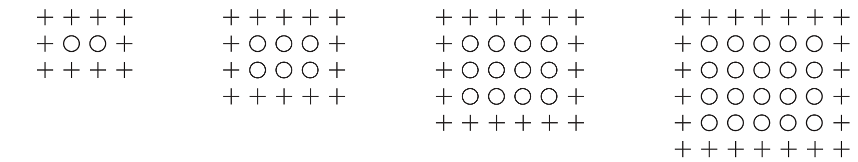
(d) A boat leaves P and sails in a straight line to Q.

(i) It takes 4 hours and 53 minutes to sail from P to Q.
It arrives at Q at 02 23.

At what time does it leave P? [1]

(ii) Calculate the shortest distance between the boat and the lighthouse. [2]

10



Flowerbed 1 Flowerbed 2 Flowerbed 3 Flowerbed 4

The diagrams above show the first four flowerbeds in a sequence.
Each flowerbed contains two types of plant, pansies (+) and primroses (○).

The table shows the number of plants in the first three flowerbeds.

Flowerbed number (n)	1	2	3	4	5
Number of pansies	10	14	18		
Number of primroses	2	6	12		
Total number of plants	12	20	30		

(a) Copy and complete the columns for flowerbeds 4 and 5. [2]

(b) Find an expression, in terms of n, for

(i) the number of pansies in flowerbed n, [1]

(ii) the number of primroses in flowerbed n. [1]

(c) **Hence** show that the total number of plants in flowerbed n can be expressed in the form

$$(n + 2)(n + 3).$$ [2]

(d) Calculate the total number of plants in flowerbed 10. [1]

(e) There are 306 plants in flowerbed k.

(i) Show that k satisfies the equation

$$k^2 + 5k - 300 = 0.$$ [2]

(ii) Solve the equation $k^2 + 5k - 300 = 0$. [2]

(iii) Hence find the number of pansies in flowerbed k. [1]

11 Answer the WHOLE of this question on a sheet of graph paper.

(a) The time taken by 140 children to run 200 metres was recorded.
The results are summarised in the table below.

Time (t seconds)	$22 \leqslant t < 24$	$24 \leqslant t < 26$	$26 \leqslant t < 31$	$31 \leqslant t < 36$	$36 \leqslant t < 46$
Frequency	12	18	42	28	40

(i) Using a scale of 1 cm to represent 2 seconds, draw a horizontal axis for time from 22 seconds to 46 seconds.
Using a scale of 1 cm to represent 1 unit, draw a vertical axis for frequency density from 0 to 9 units.

On your axes, draw a histogram to represent the information in the table. [3]

(ii) Estimate the number of children who took less than 25 seconds to run 200 metres. [1]

(iii) One child was chosen at random.

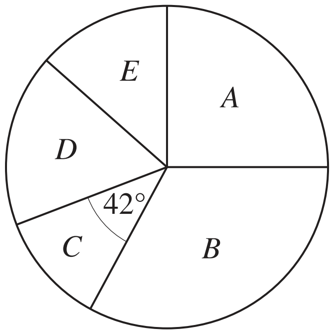
Calculate the probability that the time taken by this child was less than 36 seconds.
Express your answer as a fraction in its lowest terms. [1]

(iv) Out of the 30 children who took less than 26 seconds, two were chosen at random.

Calculate the probability that they both took less than 24 seconds. [2]

(b) Some boys were put into five groups, *A*, *B*, *C*, *D* and *E*, based on the times they took to run 100 metres.
The pie chart shows the proportion of boys in each group.

Group *A* contains $\frac{1}{4}$ of the boys.
Group *B* contains 35% of the boys.
Group *C* is represented by a sector with an angle of 42°.
Group *D* contains 9 boys.



(i) Find the fraction of boys in group *C*.
Give your answer in its lowest terms. [1]

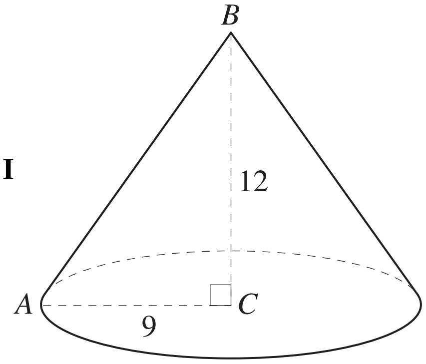
(ii) Given that the number of boys in group *B* is 21, find the total number of boys who ran the 100 metres. [2]

(iii) Calculate the number of boys in group *E*. [2]

12 **[Volume of a cone = $\frac{1}{3}\pi r^2 h$]**
[Curved surface area of a cone = πrl]

Diagram I shows a solid cone with C as the centre of its base.
B is the vertex of the cone and A is a point on the circumference of its base.
$AC = 9$ cm and $BC = 12$ cm.

Diagram I



(a) Calculate

(i) AB, [2]

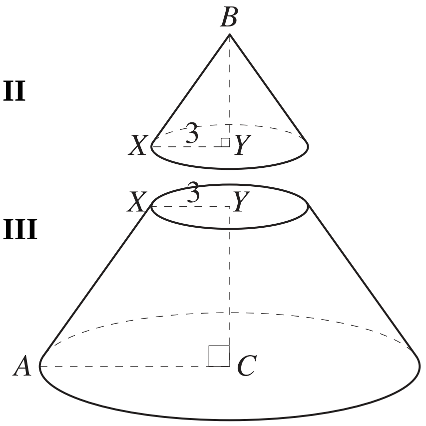
(ii) the **total** surface area of the cone, [2]

(iii) the volume of the cone. [2]

(b) The cone in Diagram I is cut, parallel to the base, to obtain a small cone shown in Diagram II and a frustum shown in Diagram III.
Y is the centre of the base of the small cone.
X is the point on the circumference of this base and on the line AB such that $XY = 3$ cm.

Diagram II

Diagram III



Calculate

(i) BY, [1]

(ii) AX, [1]

(iii) the circumference of the base of the small cone, [2]

(iv) the volume of the frustum. [2]